Oscillations

(Simple pendulum, Loaded spring & energy in SHM)

Note : This PPT will NOT help you learn physics concepts. It is intended only as a quick revision of formulas, definitions, theorems and concepts before examinations. No physics can be learnt just by watching a few videos or going through a few slides of PPT.

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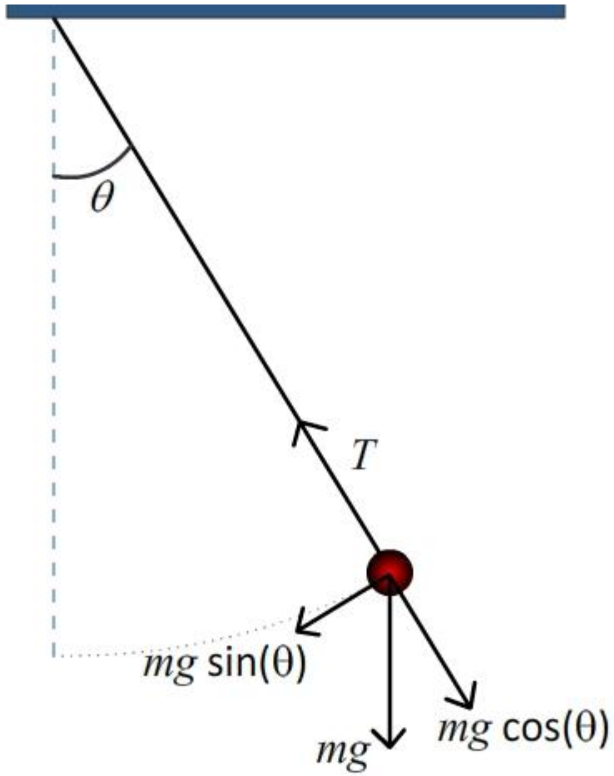
Simple pendulum

- A bob (assumed to a point mass) is suspended by a string
- The string is of length l and it is assumed to be inextensible and of negligible mass.
- Effect of other dissipative forces is assumed to be negligible.
- When the bob is displaced from its equilibrium position and released, it executes periodic, oscillatory, to and fro motion about the mean position (O)



- Resultant motion of the bob may be determined by resolving the forces acting on it into components

  (a) parallel to the string

  (b) perpendicular to the string

Simple pendulum simulation

Simple pendulum

Parallel to the string

$$T = mg\cos(\theta)$$

Perpendicular to the string

$$ma = -mg\sin(\theta)$$

$$a = -g\sin(\theta)$$

For small displacements $\sin(\theta) \approx \theta$ therefore

$$a = -g\theta \quad \text{— i}$$

Using relation between linear & angular displacements

$$x = l\theta$$

$$\theta = \frac{x}{l} \quad \text{— ii}$$

$$a = -g\left(\frac{x}{l}\right)$$

$$a = -\left(\frac{g}{l}\right)x \quad \text{— iii}$$

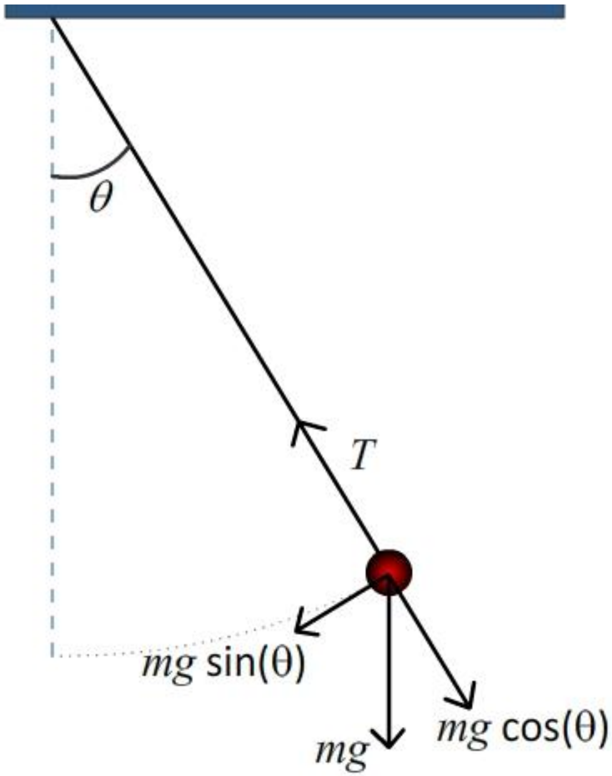
Acceleration in SHM is given by

$$a = -\omega^2 x \quad \text{— iv}$$

Comparing (iii) and (iv) we get

$$\omega^2 = \frac{g}{l}$$

$$\omega = \sqrt{\frac{g}{l}}$$



$$\Rightarrow \frac{2\pi}{T} = \sqrt{\frac{g}{l}}$$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

Simple pendulum

Seconds pendulum : A pendulum whose time period is 2 seconds is called a seconds pendulum.

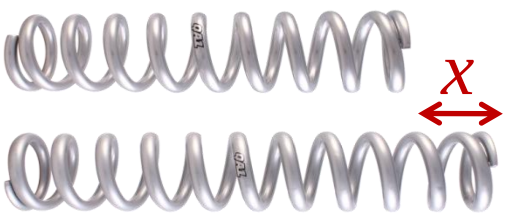
Length of a simple pendulum beating seconds is nearly 100 cm or 1m.

Simple pendulum simulation

Force exerted by a spring

When a spring is either elongated or compressed, it exerts a restoring force that is proportional to the amount of elongation or compression in it.

$$\boxed{F = -kx}$$



Negative sign indicates that direction of restoring force due to the spring is opposite to extension/compression caused in the spring.

k is spring constant (a measure of *strength of spring*).

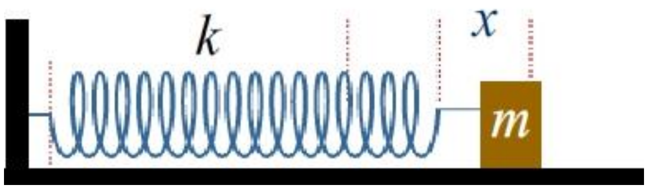
Higher value of k implies a *stronger* spring.

SI unit of k is Nm^{-1}.

Spring pendulum (loaded spring)

Consider a spring of spring constant k placed on a smooth horizontal surface. One end of the spring is fixed to a rigid support and the other end is connected to a body of mass m. The body is displaced causing an extension x in the spring, and the released.

Loaded spring simulation



Restoring force of the spring is $-kx$ therefore

$$F = -kx \quad \text{(i)}$$

From Newton's second law we get

$$F = ma \quad \text{(ii)}$$

Equating (i) and (ii) we get

$$ma = -kx$$

$$a = -\frac{k}{m}x \quad \text{(iii)}$$

The body executes SHM as acceleration is in a direction opposite to displacement and proportional to it.

Acceleration in SHM is given by

$$a = -\omega^2 x \quad \text{(iv)}$$

Comparing equations (iii) and (iv) we get

$$\omega^2 = \frac{k}{m}$$

$$\frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

Simple pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Time period is directly proportional to square root of length of pendulum (l)

Time period is inversely proportional to square root of acceleration due to gravity (g)

Time period is independent of mass of the object (m)

Time period varies with height / depth from the surface of the earth due to change in g.

Spring pendulum

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Time period is directly proportional to square root of mass of the object (m)

Time period is inversely proportional to square root of spring constant (k)

Time period is independent of acceleration due to gravity (g)

Time period does not vary with height / depth from the surface of the earth as it is independent of g.

Kinetic energy in SHM

Kinetic energy of a body of mass m having velocity v is given by

$$KE = \frac{1}{2}mv^2 \quad \text{--- (i)}$$

Velocity of a body executing SHM is given by the relation

$$v = \omega\sqrt{A^2 - y^2} \quad \text{--- (ii)}$$

Substituting equation (ii) in equation (i) we get

$$\boxed{KE = \frac{1}{2}m\omega^2\left(A^2 - y^2\right)}$$

- KE is maximum at the mean position ($y = 0$)
- $KE_{max} = \frac{1}{2} m\omega^2 A^2$
- KE is zero that the extreme positions ($y = \underline{+}A$)

Potential energy in SHM

Work done by a force F is causing a displacement dx in a body is given by

$$\mathrm{d}W = F \bullet \mathrm{d}y \quad \text{—— i}$$

Force acting on a body executing SHM is given by the relation $F = -k\,y$. Therefore work done by an external force acting on the body is given by

$$\mathrm{d}W = ky\,\mathrm{d}y \quad \text{—— ii}$$

Total work done on the body is obtained by integrating the above expression.

$$W = \int_{i}^{f} ky\,\mathrm{d}y$$

$$W = \frac{1}{2} k\,y^2$$

Since the force is conservative, this work done is stored as PE. Therefore

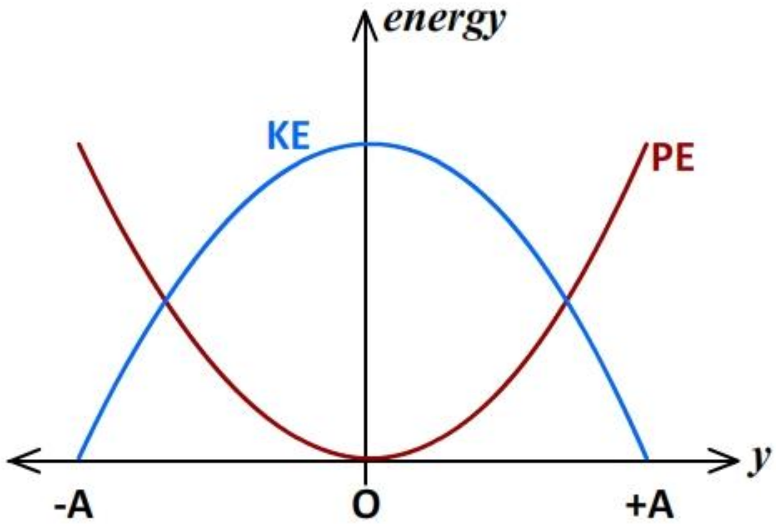
$$\boxed{PE = \frac{1}{2} k\,y^2}$$

Using $k = m\omega^2$ we get

$$\boxed{PE = \frac{1}{2} m\omega^2 y^2}$$

- PE is maximum at the extreme position ($y = \pm A$)
- PE_{max} = ½ $m\omega^2 A^2$
- PE is zero that the mean positions ($y = 0$)

Energy considerations in SHM



$$PE = \frac{1}{2} m\omega^2 y^2$$

$$PE = \frac{1}{2} m\omega^2 A^2 \sin^2(\omega t)$$

$$KE = \frac{1}{2} m\omega^2 \left(A^2 - y^2\right)$$

$$KE = \frac{1}{2} m\omega^2 A^2 \cos^2(\omega t)$$

- As the body moves away from mean position, its potential energy increases and its kinetic energy decreases.
- Total energy of the body remains constant at any position
- $TE = \frac{1}{2} m\omega^2 A^2$
- TE varies with a time period of $T/2$

Energies in SHM simulation

Comparison of parameters

Parameter	Mean position	Extreme position
Displacement (y)	Zero	$\pm A$ (Maximum)
Velocity (v)	$A\omega$ (Maximum)	zero
Acceleration (a)	Zero	$A\omega^2$ (Maximum)
Kinetic energy	$\frac{1}{2} m\omega^2 A^2$ (Maximum)	zero
Potential energy	zero	$\frac{1}{2} m\omega^2 A^2$ (Maximum)
Total energy	$\frac{1}{2} m\omega^2 A^2$	$\frac{1}{2} m\omega^2 A^2$

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